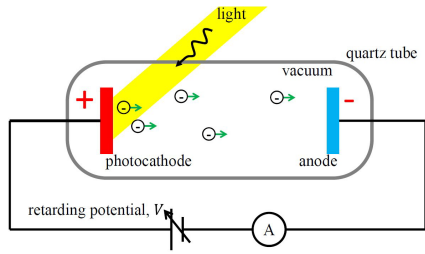


1 Wave-Particle Duality

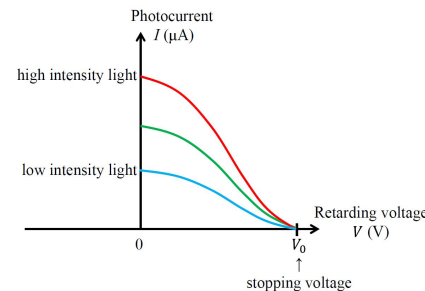
1.1 Photoelectric Effect

In this experiment, monochromatic light of a particular frequency, f , is shone upon a metal electrode (photocathode), releasing electrons from the metal with an energy, E .



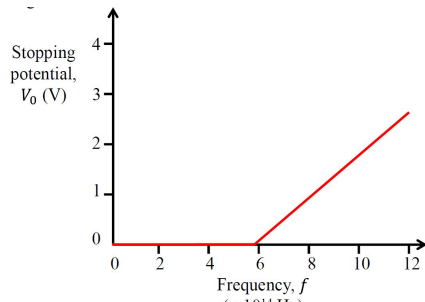
Vary light intensity and keep constant light frequency f , we get the following results:

1. Photocurrent increases linearly with increasing light intensity.
2. There is a stopping voltage, V_0 , beyond which the current stops.
3. The light intensity does not affect V_0 .



Vary light frequency f and keep constant light intensity, we get the following results:

1. Current flows only when f exceeds a certain value. i.e. A minimum f is required for any electron emission.
2. Magnitude of the stopping voltage increases linearly with f . i.e. The energy of the most energetic electrons increases linearly with f .



The gradient, h , is the same for all metal photocathodes. This is known as *Planck's constant*. The y-intercept Φ is the *work function* of the metal photocathode.

1.2 De Broglie's Postulate

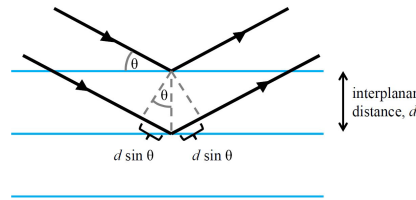
de Broglie postulated that if waves can behave like particles, then particles with mass should behave like waves with a de Broglie wavelength:

$$\lambda = \frac{h}{p}$$

The kinetic energy is given by $E = \frac{p^2}{2m}$.

1.3 Electron Diffraction

Let's consider a beam of X-rays of wavelength, λ , impinging on a regular crystal structure at an angle, θ .



Constructive interference occurs when the difference in path length is $n\lambda$, where n is an integer. A diffraction pattern will be formed with peaks when:

$$n\lambda = 2d \sin \theta$$

2 The Schrodinger equation

2.1 Wavefunctions

An electron can still be restricted to a localised region of space. The wave associated with an electron moving with a velocity v might look like a wave packet, ψ .

If the electron is being accelerated by an electric field, the position and wavelength of the wave will change with space

and time, so we can write $\psi(x, y, z, t)$ which is known as the *wavefunction* of the particle.

In terms of complex exponentials, the formula for a travelling wave is:

$$\psi(x, t) = Ae^{j(kx - \omega t)}$$

Wavenumber k is the spatial frequency of a wave. The wave packet contains a distribution of k :

$$\psi(x, t) = \sum_i A_i e^{j(k_i x - \omega_i t)}$$

2.2 The Schrodinger Equation

In essence, the Schrodinger equation is a statement of the conservation of energy $E = T + V$.

The total energy, E :

$$E\psi = j\hbar \frac{\partial \psi}{\partial t}$$

The kinetic energy, T :

$$T\psi = -\frac{\hbar^2}{2m} \frac{\partial^2 \psi}{\partial x^2}$$

The *time dependent Schrodinger equation* (TDSE) in one dimension:

$$-\frac{\hbar^2}{2m} \frac{\partial^2 \psi(x, t)}{\partial x^2} + V(x, t)\psi(x, t) = j\hbar \frac{\partial \psi(x, t)}{\partial t}$$

For the case where the potential, V , is not varying with time, and assuming that the wavefunction is separable, such that $\psi(x, t) = \Psi(x)\Theta(t)$.

The *time independent Schrodinger equation* (TISE) in one dimension:

$$-\frac{\hbar^2}{2m} \frac{\partial^2 \Psi}{\partial x^2} + V\Psi = E\Psi$$

2.3 Postulates

Basic postulates of quantum mechanics:

1. For a particle in a potential, there is an associated wavefunction, ψ which determines everything that can be known about the particle.
2. The wavefunction satisfies the Schrodinger equation at all times.
3. The wavefunction is a *complex, single-valued* function of space coordinates and time and must be *continuous*, as must $\partial\psi/\partial x$, $\partial\psi/\partial y$ and $\partial\psi/\partial z$.

4. The *probability*, $P(x, y, z)$ of finding the particle in a volume element $\partial x \partial y \partial z$ about a point (x, y, z) is:

$$P(x, y, z) = |\psi|^2 \partial x \partial y \partial z$$

2.4 The Uncertainty Principle

Heisenberg's uncertainty principle: If Δp and Δx are the uncertainties in momentum and position respectively, then:

$$\Delta x \Delta p \geq \frac{\hbar}{2}$$

To measure the wavelength with infinite accuracy, we would need an infinitely extended wavepacket. Similarly, as energy is related to frequency from $E = hf$, and a sufficient length of time is required to accurately measure a frequency:

$$\Delta E \Delta t \geq \frac{\hbar}{2}$$

2.5 Particle in an Infinite Potential Well

We need to find a solution to the TISE with the potential term given by:

$$\begin{cases} V = 0 & \text{for } 0 < x < L \\ V = \infty & \text{for } x < 0, x > L \end{cases}$$

Outside the well, $V = \infty$, and the TISE only physically realistic solution $\Psi(x) = 0$. Inside the well, the TISE has the general solution $\Psi = A \cos(kx) + B \sin(kx)$.

For the wavefunction to be continuous and single-valued, the wavefunctions for inside and outside the boundary must be identical at the boundary. The total probability:

$$\int_0^L |\Psi|^2 dx = 1$$

The *normalised solution*:

$$\Psi = \sqrt{\frac{2}{L}} \sin\left(\frac{\pi n}{L} x\right)$$

We can see that quantum mechanics only permits the electron to exist in a limited number of states. n is known as a *quantum number* and it defines the state uniquely.

The energy of the electron in any state is:

$$E = \frac{\pi^2 n^2 \hbar^2}{2mL^2}$$

Energy is *quantised* and is never zero. The lowest possible energy state, associated with $n = 1$, is known as the *ground state*.

3 The Hydrogen Atom

3.1 The TISE in Three Dimensions

From Gauss' law of electrostatics, we know the electrostatic potential, V , varies with distance, r , from the proton as:

$$V(r) = \frac{-e^2}{4\pi\epsilon_0 r}$$

In 3D the TISE becomes:

$$-\frac{\hbar^2}{2m} \nabla^2 \Psi + V\Psi = E\Psi$$

The wavefunction, $\Psi(r, \theta, \phi)$, may be separated into functions of r, θ and ϕ :

$$\Psi(r, \theta, \phi) = R(r)\Theta(\theta)\Phi(\phi)$$

The $\Phi(\phi)$ component has the form:

$$\Phi(\phi) = Ae^{jm_l \phi}$$

m_l is an integer called the *magnetic quantum number*.

The $\Theta(\theta)$ component is the set of *Legendre functions*:

$$\Theta(\theta) = P_{l, m_l}(\theta)$$

l is another quantum number, known as the *azimuthal quantum number* or *orbital angular momentum quantum number*. l must be a positive integer and $|m_l| \leq l$.

The magnitude of the electron's orbital angular momentum, L , is related to l by:

$$L = \sqrt{l(l+1)}\hbar$$

The component of the electron's orbital angular momentum along the z direction, L_z , is related to m_l by:

$$L_z = m_l \hbar$$

The $R(r)$ component, the radial component, has the form:

$$R(r) = \frac{f_{n, l}(r)}{r} \exp\left(\frac{-r}{a}\right)$$

$f_{n, l}(r)$ is a polynomial function of r whose form depends on l and a new quantum number, n , known as the *principal quantum number*. n is a positive integer and $n > l$.

For the one electron system here, it is the value of n alone which determines the energy of the electron:

$$E_n = \frac{-e^4 m}{8\epsilon_0^2 \hbar^2 n^2}$$

3.2 Pauli Exclusion Principle

The Pauli exclusion principle states that: In a multielectron system there can never be more than one electron in the same state.

Electrons actually have a fourth quantum number, m_s , which is known as spin. For electrons:

$$m_s = \pm \frac{1}{2}$$

The magnitude, S , of the spin angular momentum of the electron is given by:

$$S = \sqrt{\frac{1}{2} \left(1 + \frac{1}{2}\right)} \hbar = 0.87 \hbar$$

The component of the spin angular momentum in the z direction, S_z , is determined by m_s :

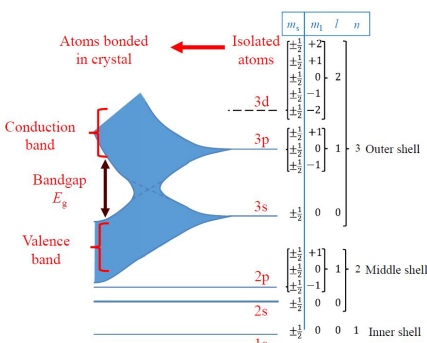
$$S_z = m_s \hbar = \pm \frac{1}{2} \hbar$$

4 Electron Energy Bands

4.1 Energy Band Formation

If the two Si atoms are brought close together such that the two $3p$ wavefunctions overlap, then the Pauli exclusion principle is invoked which states that the two electrons cannot exist in the same $3p$ state.

Therefore, the two wavefunctions become perturbed and their two energy levels split, thereby making them distinct states.



An sp^3 hybridisation is said to have occurred.

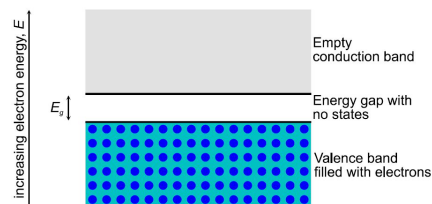
The two s states and six p states have become four bonding states, which are completely filled with the Si atom's four electrons that take part in covalent bonding

with its four neighbours, and four empty antibonding states.

The full band of bonding states is known as the valence band (VB) and the empty band of antibonding states is known as the conduction band (CB).

4.2 Band Structure and Conduction

Consider the band structure of bulk Si, which is a semiconductor.



At room temperature, the energy gap between the top of the VB and the bottom of the CB, E_g , is small enough for some electrons to be excited into the antibonding CB states.

The electron has escaped from the bond and is free to conduct. The vacancy in the VB can also contribute to conduction. We have created an electron-hole pair.

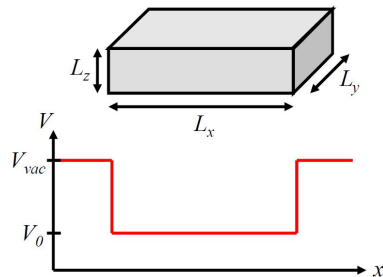
For a pure semiconductor (intrinsic semiconductor) the number of electrons in the CB, n , and holes in the VB, p , must be equal at any time. The intrinsic carrier concentration $n_i = n = p$.

In an insulator the CB and VB are separated by an energy gap which is large. So, an insignificant number of electrons are excited into the CB even at room temperature.

In the case of metals, either the bands overlap or are partially filled. Electrons can move into new states with essentially no change in energy and take part in conduction. The conductivity will decrease with temperature as rising lattice vibrations act to increase the scattering of electrons moving through the crystal lattice.

4.3 Free Electron Model

We assume a number of electrons moving independently in a square well potential.



The solution to the TISE is a plane wave:

$$\Psi = A \exp(j\mathbf{k} \cdot \mathbf{r}) \exp(-j\omega t)$$

$\mathbf{k} = (k_x, k_y, k_z)$ is the wavevector of the electron and is related to its momentum, \mathbf{p} , using the de Broglie postulate:

$$\mathbf{p} = \hbar \mathbf{k}$$

The wavenumber, k , is the magnitude of the wavevector, \mathbf{k} :

$$k = \sqrt{k_x^2 + k_y^2 + k_z^2}$$

Substituting these boundary conditions into $\Psi = A \exp(j\mathbf{k} \cdot \mathbf{r})$ imposes the condition:

$$k_x = \pm \frac{2\pi}{L_x} n_x \quad k_y = \pm \frac{2\pi}{L_y} n_y \quad k_z = \pm \frac{2\pi}{L_z} n_z$$

Integers n_x, n_y and n_z , are analogous to the principal quantum number, n .

We can map out these wavevectors as a grid of points in \mathbf{k} -space. An electron can exist in any one of a number of discrete states in \mathbf{k} -space. This is also known as momentum space because $\mathbf{p} = \hbar \mathbf{k}$.

The energy of the electron in any state is:

$$E = \frac{\hbar^2 |\mathbf{k}|^2}{2m} = \frac{\hbar^2 k^2}{2m}$$

Wavevectors of the same magnitude have the same energy, forming spherical shells.

4.4 The Density of States

Each time we increase n_x, n_y or n_z by one, we obtain a new point in \mathbf{k} -space. We can have two electrons at each point, so the density of states in \mathbf{k} -space is:

$$\frac{2L_x L_y L_z}{8\pi^3} = \frac{2V}{8\pi^3}$$

The number of states within a spherical shell with $|\mathbf{k}|$ between k and $k + dk$ is:

$$n(k) dk = \frac{2V}{8\pi^3} 4\pi k^2 dk = \frac{V}{\pi^2} k^2 dk$$

The density of states as a function of energy, E :

$$g(E) dE = \frac{V}{2\pi^2 \hbar^3} (2m)^{\frac{3}{2}} E^{\frac{1}{2}} dE$$

4.5 Fermi-Dirac Statistics

At absolute zero, $T = 0K$, the N states with the lowest energies will be occupied. The energy of the highest filled state is called the Fermi level, E_F .

Therefore, the probability that a state is occupied depends on whether it lies above or below the Fermi level:

$$f(E) = \begin{cases} 1, & E < E_F \\ 0, & E \geq E_F \end{cases}$$

N states will be occupied:

$$N = \int_0^{E_F} g(E) f(E) dE = \int_0^{E_F} g(E) dE$$

The Fermi level, E_F , can be evaluated knowing N :

$$E_F = \frac{\hbar^2}{2m} \left(3\pi^2 \frac{N}{V} \right)^{\frac{2}{3}}$$

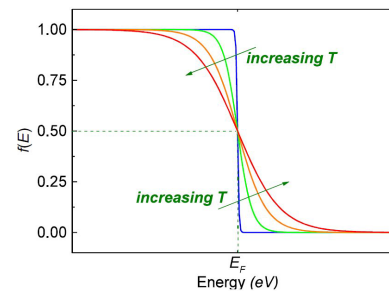
As the temperature rises above absolute zero, some electrons within an energy $\sim kT$ of the Fermi level have sufficient energy to enter states above E_F .

The number of particles, n , with a particular energy, E , is determined by the Boltzmann distribution:

$$n \propto \exp\left(\frac{-E}{kT}\right)$$

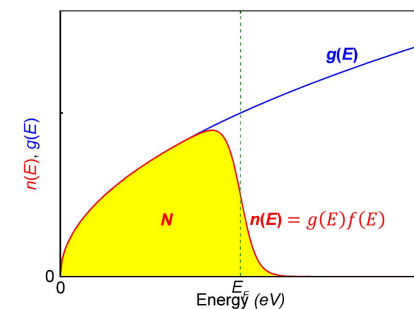
The probability that a state is occupied is given by the Fermi function, $f(E)$:

$$f(E) = \frac{1}{\exp\left(\frac{E-E_F}{kT}\right) + 1}$$



The Fermi level, E_F , marks the energy at which there is a 50% probability that a state is occupied. At absolute zero, the Fermi level is also called the Fermi energy, E_F .

The number of electrons, n , with a particular energy, E , is the product of the density of states and the Fermi function.

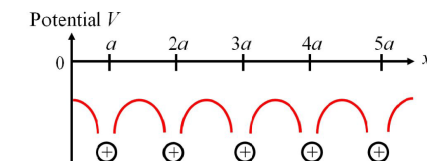


The total number of free electrons, N , is then:

$$N = \int_0^{E_F} n(E) dE = \int_0^{E_F} g(E) f(E) dE$$

4.6 Nearly Free Electron Model

Consider a simple situation in one dimension where atoms have a spacing a :



Consider a plane wave in one dimension:

$$\psi = A \exp(jkx) \exp(-j\omega t)$$

We know from optical systems, that a wave can pass through a diffraction grating of spacing a , unless a is an integer, n , number of half wavelengths (Bragg's Law with $\theta = 90^\circ$):

$$a = \frac{n\lambda}{2}$$

If the wavelength of the electron does not meet the Bragg reflection criteria, then the electron will propagate as a travelling wave through the crystal.

If the wavelength of the electron does meet the Bragg reflection criteria, then it

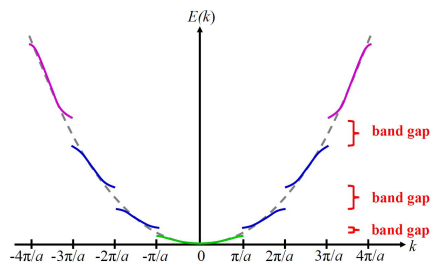
will be Bragg-reflected and forms a *standing wave*.
In terms of the wavenumber, Bragg reflection occurs when:

$$k = \pm \frac{n\pi}{a}$$

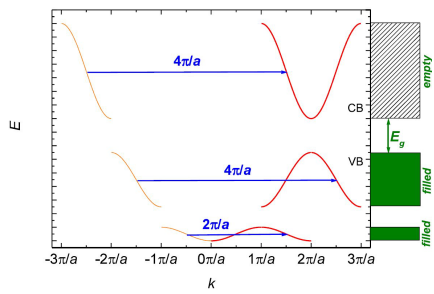
There are two possible scenarios:

1. There is a *low* probability of the electron being near the nuclei, outside the potential well associated with any one nucleus, and such a state will have a *higher energy*, $E = E_{\text{free}} + \Delta E$ associated with it than the free electron theory predicts.
2. There is a *high* probability of the electron being near the nuclei, in the bottom of the potential wells, and such a state will have a lower energy, $E = E_{\text{free}} - \Delta E$, associated with it.

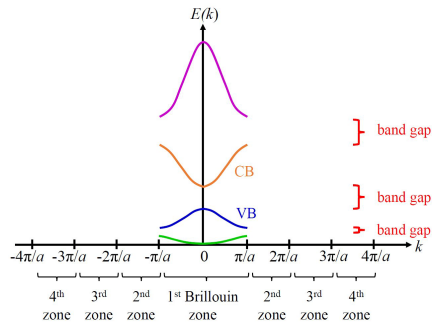
Therefore, there is an energy gap, $E_g = 2\Delta E$ between the two states.
The effect of the periodic lattice has been to open up periodic energy gaps (*band gaps*) in the $E-k$ diagram.



Block's theorem: The wavefunction of an electron in a periodic potential is a plane wave times a function with the periodicity of the lattice.
It is impossible to distinguish between an electron with a wavenumber k and one with a wavenumber $k + n2\pi/a$ where n is an integer. We can move any point on an $E-k$ diagram by $\pm n2\pi/a$ along the k axis without changing the physical significance as the two states are equivalent.



Condense all the information into the first Brillouin zone:



Semiconductors where the bottom of the conduction band has the same k as the top of the valence band are known as *direct gap* semiconductors an electron only requires an change in energy to make a transition between the two.

More commonly, semiconductors are *indirect* where the conduction band minimum occurs at a different k to the valence band maximum. To transition from the VB to the CB, an electron will require energy E_g and the addition of momentum.

4.7 Effective Mass

An electron in a periodic potential behaves differently to a free electron. The change in response is accounted for by the concept of an *effective mass*.

In terms of wave theory, the electron's velocity is given by

$$v = \frac{\partial \omega}{\partial k} = \frac{1}{\hbar} \frac{\partial E}{\partial k}$$

This defines the *group velocity* of a wave.

$$F = m^* \frac{dv}{dt} = \frac{dp}{dt}$$

The effective mass of the electron can be

found from this expression:

$$m^* = \hbar^2 \left(\frac{d^2 E}{dk^2} \right)^{-1}$$

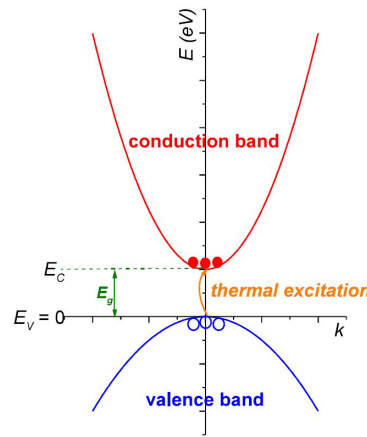
5 Electrons and Holes

5.1 The Simplified Band Diagram

We can ignore all permanently full bands below the VB and all permanently empty bands above the CB.

There are three ways of transferring electrons into the CB:

1. *Thermal excitation:* Some electrons attain enough thermal energy according to the Fermi Dirac distribution function.
2. *Atomic substitution:* An atom is substituted for one of a different element. This is known as *doping*.
3. *Photoexcitation:* Absorption of a photon (i.e. electromagnetic radiation) provides energy.



The energy of the gap is:

$$E_g = E_C - E_V$$

5.2 Holes

If we remove a single electron from the VB with a wavenumber k_e then an empty state is left behind in the VB with wavenumber $-k_e$. Therefore, we can consider this absence of an electron to be a *hole* state of wavenumber $k_h = -k_e$.

If the energy of the electron occupying the state before removal was E_e , then the

energy of the hole, $E_h = -E_e$. Hence, the group velocity of the hole, v_h , is:

$$v_h = \frac{1}{\hbar} \frac{\partial E_h}{\partial k_h} = \frac{1}{\hbar} \frac{\partial (-E_e)}{\partial (-k_e)} = v_e$$

The effective mass of the hole is:

$$m_h = \hbar^2 \left(\frac{d^2 E_h}{dk_h^2} \right)^{-1} = -m_e$$

The application of an electric field, ϵ , will cause a change in hole momentum, p_h .

$$F = \hbar \frac{dk_h}{dt} = -(-e\epsilon) = e\epsilon$$

The hole behaves as though it has a positive charge of $+e$.

5.3 Charge Carrier Populations

Two important parameters in the electrical characterisation of a semiconductor are the number density of electrons in the CB, n , and the number density of holes in the VB, p .

Density of states in the CB:

$$g(E)dE = \frac{V}{2\pi^2 \hbar^3} (2m_e^*)^{\frac{3}{2}} (E - E_C)^{\frac{1}{2}} dE$$

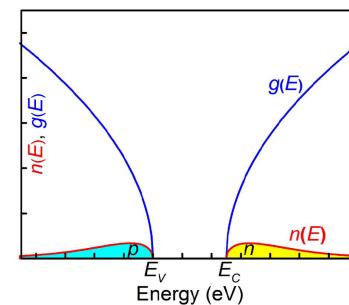
Density of states in the VB:

$$g(E)dE = \frac{V}{2\pi^2 \hbar^3} (2m_h^*)^{\frac{3}{2}} (E_V - E)^{\frac{1}{2}} dE$$

The probability, $f(E)$, that a state is occupied by an electron is given by the Fermi function.

The probability, $f_p(E)$, that a state is occupied by a hole, is $1 - f(E)$:

$$f_p(E) = 1 - \frac{1}{e^{\left(\frac{E-E_F}{kT}\right)+1}} = \frac{1}{e^{\left(\frac{-E-E_F}{kT}\right)+1}}$$



If $E_C - E_F \gg kT$, occupancy of the CB is low. Therefore, the Fermi function can be approximated by the Boltzmann distribution:

$$f(E) = e^{\left(\frac{-E-E_F}{kT}\right)}$$

The number density of electrons in the CB can be written as:

$$n = N_C e^{\left(\frac{E_F - E_C}{kT}\right)}$$

N_C is known as the *effective density of states* in the conduction band.

$$N_C = 2 \left(\frac{m_e^* kT}{2\pi \hbar^2} \right)^{\frac{3}{2}}$$

The number density of holes in the VB can be written as:

$$p = N_V e^{\left(\frac{-E_F - E_V}{kT}\right)}$$

N_V is known as the *effective density of states* in the valence band.

$$N_V = 2 \left(\frac{m_h^* kT}{2\pi \hbar^2} \right)^{\frac{3}{2}}$$

5.4 The Law of Mass Action

The product of the electron and hole densities is a constant:

$$np = N_C N_V e^{\left(\frac{-E_g}{kT}\right)} = n_i^2$$

$E_g = E_C - E_V$ is the band gap energy and n_i is known as the *intrinsic carrier density*.

$$n_i = \sqrt{N_C N_V} e^{\left(\frac{-E_g}{2kT}\right)}$$

The intrinsic carrier density, n_i , is a property of a given semiconductor material.

5.5 Intrinsic Semiconductor

If we have a pure (*intrinsic*) semiconductor, we know that we can only create an electron in the CB by creating a hole in the VB.

$$n = p = n_i = \sqrt{N_C N_V} e^{\left(\frac{-E_g}{2kT}\right)}$$

Let E_{Fi} denote the Fermi level in an intrinsic material, the *intrinsic Fermi level*.

$$E_{Fi} = \frac{E_C + E_V}{2} + \frac{3kT}{4} \ln \left(\frac{m_h^*}{m_e^*} \right)$$

Usually $E_C + E_V \gg kT$ and so for intrinsic material:

$$E_{Fi} = \frac{E_C + E_V}{2}$$

5.6 Charge Carrier Mobility

There is a linear relationship between carrier drift velocity, v , and electric field:

$$v = \mu \epsilon$$

The constant of proportionality, μ , known as the *carrier mobility*, is:

$$\mu = \frac{q\tau}{m^*}$$

Mobility has units of $\text{m}^2 \text{V}^{-1} \text{s}^{-1}$, and is related to the current density from Ohm's Law according to:

$$J = nq\langle v \rangle = nq\mu \epsilon = \sigma \epsilon$$

The conductivity, σ , is therefore directly dependent on the carrier mobility by:

$$\sigma = nq\mu = n \frac{q^2 \tau}{m^*}$$

For a semiconductor with both electrons and holes:

$$\sigma = ne\mu_e + pe\mu_h$$

For an intrinsic semiconductor:

$$\sigma = e(\mu_e + \mu_h) \sqrt{N_C N_V} e^{\left(\frac{-E_g}{2kT}\right)}$$

6 Doping

6.1 Extrinsic Semiconductor

The type of conduction (n -type or p -type) can be controlled by deliberately adding tiny amounts of impurities into the crystal lattice.

The impurity atoms are known as *dopants* and the addition of dopants is known as *doping*. Semiconductors that have been doped are called *extrinsic*.

There are two types of dopants:

1. *Donors* are impurity atoms used to achieve n -type doping.
2. *Acceptors* are impurity atoms used to achieve p -type doping.

6.2 Doping

The aim of n -type doping is to increase the number of electrons in the CB.

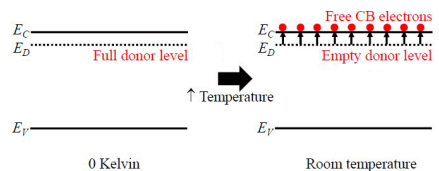
Silicon is a group IV element ($1s^2 2s^2 2p^6 3s^2 3p^2$).

1. To n -dope, we add a donor dopant impurity from group V: phosphorous ($1s^2 2s^2 2p^6 3s^2 3p^3$), arsenic or antimony.

2. To p -dope, we add an acceptor impurity from group III: boron, aluminium.

Group V impurities generate an energy level, known as the donor level, E_D , in the gap.

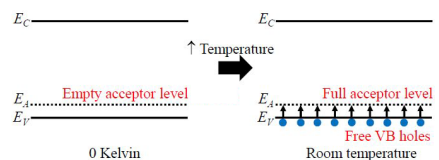
The acceptor level (E_A) will be empty at 0 K. As the temperature rises, electrons are easily excited to the acceptor level (E_A) from the VB.



The donor donates electrons to the CB without the creation of a hole in the VB. Therefore this doping achieves an n -type semiconductor.

Group III impurities generate an energy level, known as the acceptor level, E_A , in the gap.

The acceptor level (E_A) will be empty at 0 K. As the temperature rises, electrons are easily excited to the acceptor level (E_A) from the VB.



The removal of the electron from the VB creates a vacant VB state for electrons to jump into: the hole. This excitation occurs without the creation of an electron in the CB. Therefore this doping achieves a p -type semiconductor.

6.3 Majority and Minority Carriers

In n -type material the electrons are called majority carriers and the holes are called minority carriers.

$$(p = n_i^2/n)$$

In p -type material the holes are called majority carriers and the electrons are

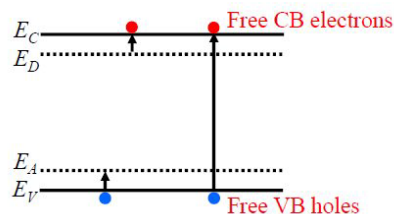
called minority carriers.

$$(n = n_i^2/p)$$

6.4 Doping in III-V Semiconductors

We can dope III-V semiconductors such as GaAs using transition metals and group IV elements. These act as p -type or n -type dopants depending on their chemical preference for substituting for the Ga or the As.

At 0 K, all donor levels will be occupied and no e^- will be in the CB while all acceptor levels will be empty with no holes in the VB. As T increases, e^- in the donor level enter the CB and e^- in the VB can enter the acceptor level. At high T , e^- can get all the way from the VB into the CB.



We define:

- N_A density of acceptor atoms
- N_D density of donor atoms
- n_D density of unionised donor atoms (no charge)
- p_A density of unionised acceptor atoms (no charge)

When a donor is ionised, it loses an electron and becomes positively charged. The semiconductor remains electrically neutral.

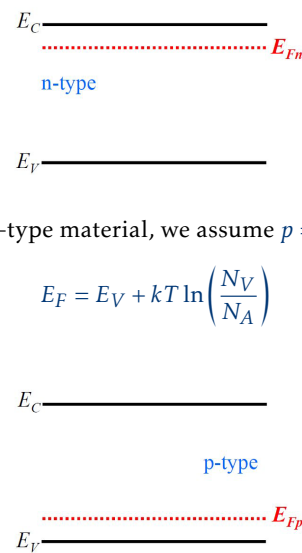
When an acceptor is ionised, it gains an electron and becomes negatively charged. The semiconductor remains electrically neutral.

$$p + (N_D - n_D) = n + (N_A - p_A)$$

At room temperature, we can assume all donors will be ionised $n_D = p_A = 0$.

For n -type material, we assume $n = N_D$:

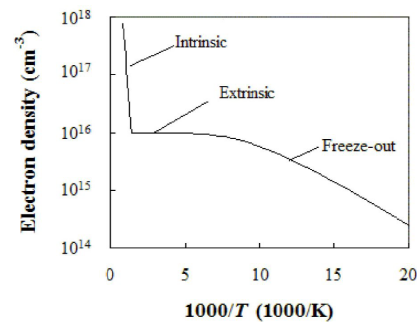
$$E_F = E_C - kT \ln\left(\frac{N_C}{N_D}\right)$$



For p -type material, we assume $p = N_A$:

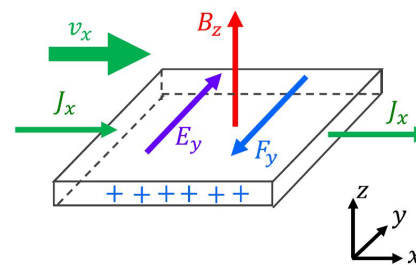
$$E_F = E_V + kT \ln\left(\frac{N_V}{N_A}\right)$$

Temperature dependence of charge carrier distribution:



6.5 Hall Effect

For p -type semiconductors, q is positive for holes and *hole current* occurs in the same direction as classical current, $J_x = pev_x$.



At equilibrium, the force which the electric field E_y exerts on the carriers is equal

and opposite to the force exerted by the magnetic field B_z .

$$qE_y = qv_x B_z$$

Therefore a voltage V_H appears across the width of material w :

$$V_H = E_y w$$

The Hall coefficient, R_H , is defined as:

$$R_H = \frac{E_y}{J_x B_z} = \frac{1}{pe}$$

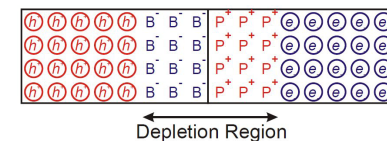
For n -type material, the polarity of V_H is reversed and:

$$R_H = -\frac{1}{ne}$$

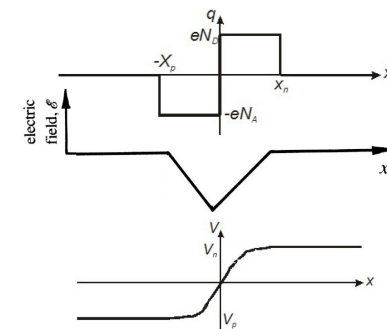
7 The pn Junction Diode

7.1 Junction Structure

This diffusion of electrons and holes rapidly lowers the amount of mobile charge on either side of the junction to leave a zone which is depleted of mobile carriers but has a background of ionised impurity atoms known as the *depletion region*.



The charge imbalance caused by these ionised impurity atoms sets up a potential difference which eventually prevents further $e-h$ annihilation, and an equilibrium is reached.

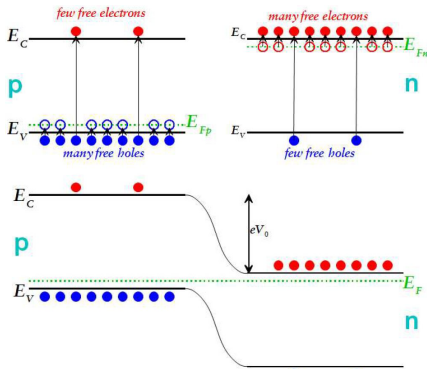


7.2 Built In Potential

Abrupt junction model:

1. The junction is *abrupt*.
2. *Recombination* in the junction is negligible.
3. No fields exist outside the junction.

The depletion region is electrically charged and the charge adjusts (by a change in the depletion region width) to align the *p*-type and *n*-type Fermi levels.



The equilibrium potential difference (*contact potential*):

$$eV_0 = E_{fn} - E_{fp} = kT \ln \left(\frac{N_A N_D}{n_i^2} \right)$$

7.3 Depletion Region Width

The *charge neutrality* equation:

$$eN_D x_n = eN_A (-x_p)$$

x_n and $-x_p$ are the widths of the depletion regions on the *n* and *p* sides, respectively.

The Poisson equation states that:

$$\nabla^2 V = \frac{-\rho}{\epsilon_0 \epsilon_r} = \frac{eN_A}{\epsilon_0 \epsilon_r}$$

The total barrier voltage (barrier potential) is:

$$V_0 = \frac{e}{2\epsilon_0 \epsilon_r} (N_A x_p^2 + N_D x_n^2)$$

The total width of the junction:

$$w = x_n - x_p = \left(\frac{2\epsilon_0 \epsilon_r V_0}{e} \right)^{\frac{1}{2}} \left[\left(\frac{N_A}{N_A N_D + N_D^2} \right)^{\frac{1}{2}} + \left(\frac{N_D}{N_A N_D + N_A^2} \right)^{\frac{1}{2}} \right]$$

Given $N_A \gg N_D$ the equation can be simplified to:

$$w = \left(\frac{2\epsilon_0 \epsilon_r V_0}{eN_D} \right)^{1/2}$$

7.4 Junction Capacitance

The charge on the *n*-side is:

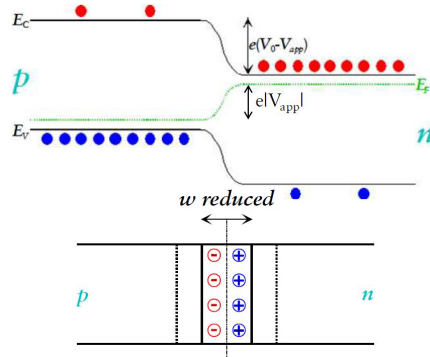
$$Q = eN_D x_n = (2\epsilon_0 \epsilon_r eV_0)^{\frac{1}{2}} \left(\frac{N_A N_D}{N_A + N_D} \right)^{\frac{1}{2}}$$

The depletion region of the *p-n* junction contains no mobile carriers and acts as a capacitor.

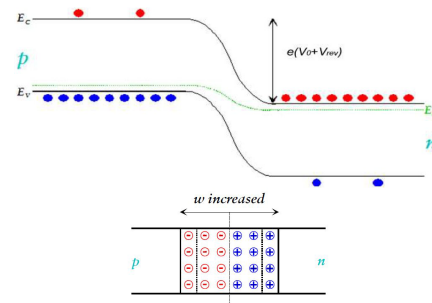
$$C_d = \frac{dQ}{dV} = \left(\frac{\epsilon_0 \epsilon_r e N_A N_D}{2V_0 (N_A + N_D)} \right)^{\frac{1}{2}} = \frac{\epsilon_0 \epsilon_r}{w}$$

7.5 Application of a Bias

If we apply a positive voltage to the *p*-side of a *p-n* junction then the energy of the electrons on the *n*-side increases while the energy of the electrons on the *p*-side decreases.



If we apply a negative voltage to the *p*-side of a *p-n* junction then the energy of the electrons on the *n*-side decreases while the energy of the electrons on the *p*-side increases.



The width w of the depletion region under the externally applied bias V_{app} :

$$w = \left(\frac{2\epsilon_0 \epsilon_r (V_0 - V_{app})}{e} \right)^{\frac{1}{2}} \left[\left(\frac{N_A}{N_A N_D + N_D^2} \right)^{\frac{1}{2}} + \left(\frac{N_D}{N_A N_D + N_A^2} \right)^{\frac{1}{2}} \right]$$

The capacitance of the biased junction:

$$C_d = \left(\frac{\epsilon_0 \epsilon_r e N_A N_D}{2(V_0 - V_{app})(N_A + N_D)} \right)^{\frac{1}{2}}$$

This is known as a *varactor*, as the capacitance is dependent on the applied voltage.

7.6 Inhomogeneous Semiconductors

If excess carriers are introduced locally, there will be a non-uniform distribution of carriers and such materials are known as *inhomogeneous semiconductors*.

Einstein Relation:

$$\frac{D_e}{\mu_e} = \frac{D_h}{\mu_h} = \left(\frac{kT}{e} \right)$$

7.7 Continuity Equation

The separate effects of *carrier generation*, *recombination*, *diffusion* and *drift* can be conveniently summarised in the equation known as the *Continuity (Master) Equation*.

The total rate of change in the excess electron concentration in the volume is given by:

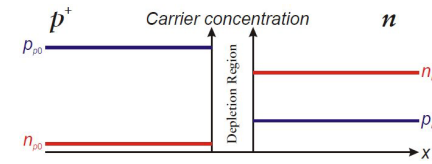
$$\begin{aligned} \frac{\partial(\Delta n)}{\partial t} &= -\frac{\Delta n}{\tau_e} + \mu_e \epsilon \frac{\partial(\Delta n)}{\partial x} + D_e \frac{\partial^2(\Delta n)}{\partial x^2} \\ &= -\frac{\Delta n}{\tau_e} + \frac{1}{e} \frac{\partial}{\partial x} J_e \end{aligned}$$

This is the continuity equation for electrons, and for holes the corresponding equation is:

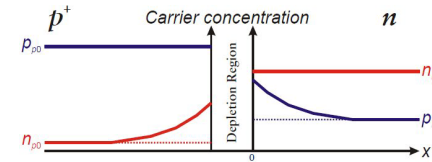
$$\begin{aligned} \frac{\partial(\Delta p)}{\partial t} &= -\frac{\Delta p}{\tau_h} - \mu_h \epsilon \frac{\partial(\Delta p)}{\partial x} + D_h \frac{\partial^2(\Delta p)}{\partial x^2} \\ &= -\frac{\Delta p}{\tau_h} - \frac{1}{e} \frac{\partial}{\partial x} J_h \end{aligned}$$

7.8 Current Flow

Consider the current flow in a *p+n* junction in which the *p*-side is more heavily doped than the *n*-side.



Consider a *p+n* junction where most of the current is carried by holes in the *n*-type semiconductor. By applying a forward bias, we lower the barrier potential to allow more holes to surmount the barrier.



The injected minority carrier concentration of holes at position $x = 0$ is:

$$\begin{aligned} p(0) &= N_A \exp \left\{ \frac{-e(V_0 - V)}{kT} \right\} \\ &= p_{n0} \exp \left(\frac{eV}{kT} \right) \end{aligned}$$

The current flow associated with a junction is limited by how these minority carriers are removed from the edge of the depletion region.

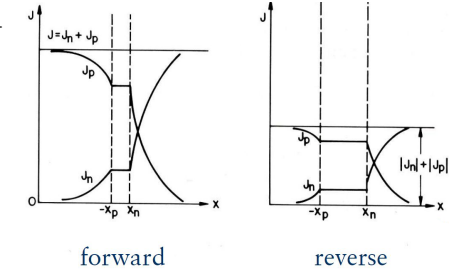
We will make the following assumptions:

1. The injected minority carrier concentration is much less than the majority carrier concentration (the low injection case).
2. No recombination takes place in the depletion region.
3. There are negligible fields outside the depletion region.

The *carrier diffusion length* $L_h = (D_h \tau_h)^{1/2}$.

$$p - p_{n0} = p_{n0} \left[\exp \left(\frac{eV}{kT} \right) - 1 \right] \exp \left(\frac{-x}{L_h} \right)$$

The total current is given by the sum of the electron and hole currents:



Minority carrier injection: Forward current carried by holes in the *n*-type semiconductor. By applying a forward bias, we lower the barrier potential to allow more holes to surmount the barrier.

Minority carrier extraction: Reverse saturation current due to drift of minority carriers over depletion region.

The ideal diode law (Shockley equation):

$$I = I_S \left[\exp \left(\frac{eV}{kT} \right) - 1 \right]$$

I_S can be written in terms of the hole lifetime of the diffusion coefficient:

$$\begin{aligned} I_S &= \text{area} \left[e \left(\frac{D_e}{L_e} \right) \frac{n_i^2}{N_A} + e \left(\frac{D_h}{L_h} \right) \frac{n_i^2}{N_D} \right] \\ &= \text{area} \left[e \left(\frac{L_e}{\tau_e} \right) n_{p0} + e \left(\frac{L_h}{\tau_h} \right) p_{n0} \right] \end{aligned}$$

For the *p+n* junction where $N_A \gg N_D$, the prefactor I_S becomes:

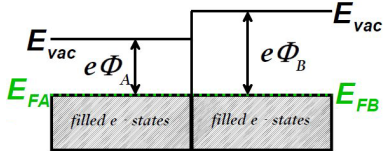
$$I_S = \text{area} \left[e \left(\frac{D_h}{L_h} \right) \frac{n_i^2}{N_D} \right]$$

8 Metal-Semiconductor Junctions

8.1 Metal-Metal Junctions

Consider two metals, *A* and *B* with different Fermi levels (E_{FA} , E_{FB}) and different work functions (the difference between the Fermi level and the vacuum level) Φ_A and Φ_B .

The *contact potential* $eV_a = \Phi_A - \Phi_B$ will exist across the junction:



8.2 Metal-Semiconductor Junctions

The relative magnitudes of the work functions of the metal and semiconductor are Φ_m and Φ_{sc} respectively. For an n -type semiconductor in contact with a metal:

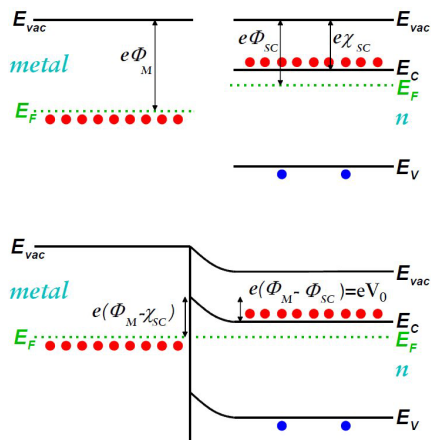
1. If $\Phi_m > \Phi_{sc}$ then we get a rectifying contact called a *Schottky Barrier* (conduction only occurs when a positive bias is applied to the metal with respect to the semiconductor)
2. If $\Phi_m < \Phi_{sc}$ then we get an *Ohmic contact* (conduction occurs for both senses of bias)

For an p -type semiconductor in contact with a metal:

1. If $\Phi_m < \Phi_{sc}$ then we get a Schottky Barrier (negative bias on the metal allows conduction)
2. If $\Phi_m > \Phi_{sc}$ then we get an Ohmic contact (conduction occurs for both senses of bias)

8.3 Schottky Barrier

For an n -type semiconductor-metal junction, a rectifying Schottky barrier will be formed if $\Phi_m > \Phi_{sc}$. The transfer of electrons from the semiconductor to the metal will continue until the Fermi levels align. A *depletion region* has formed.



The term χ_{sc} is known as the *electron affinity*, and it is the energy required by an electron at the bottom of the conduction band to escape from the semiconductor into the vacuum

$$e\Phi_{SC} = e\chi_{SC} + (E_C - E_F)$$

The contact potential is:

$$V_0 = \frac{eN_D w^2}{2\epsilon_0 \epsilon_r} = \frac{\Phi_M - \Phi_{SC}}{e}$$

The depletion width is:

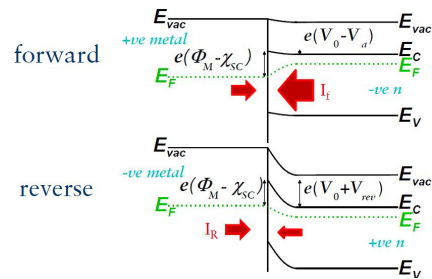
$$w = \left(\frac{2\epsilon_0 \epsilon_r (\Phi_M - \Phi_{SC})}{e^2 N_D} \right)^{1/2}$$

We can also find the capacitance per unit area:

$$C = \left(\frac{\epsilon_0 \epsilon_r e N_D}{2V} \right)^{1/2} = \frac{\epsilon_0 \epsilon_r}{w}$$

8.4 Biased Junction

The potential barrier at the junction acts as a rectifier. The barrier is $e(V_0 - V_a)$ where V_a is the *forward bias* applied to the metal.

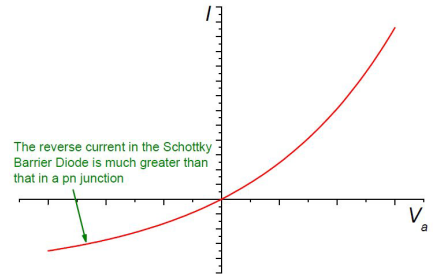


8.5 Current Flow in a Schottky Barrier

When a negative bias is applied to the metal in a Schottky Barrier Diode:

1. The forward current I_F is reduced as the barrier to electron flow from the semiconductor to the metal is increased.
2. The reverse current I_R remains the same as the barrier to electron flow from the metal to the semiconductor remains $e(\Phi_m - \chi_{sc})$.

The net current flow across the junction is $I = I_F - I_R$.



The limit to current flow is now determined by how fast the electrons can get from the semiconductor to the metal.

The Schottky barrier diode is a *unipolar* device.

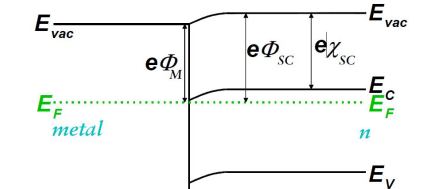
Thermionic Emission Theory: Assume that the mobility of the electrons is such that they can get across the barrier as fast as they are emitted.

The net current from the semiconductor to the metal is:

$$J = A^* T^2 \exp\left(\frac{-e\Phi_b}{kT}\right) \left[\exp\left(\frac{eV_a}{kT}\right) - 1 \right]$$

8.6 Ohmic Contacts

For an n -type semiconductor-metal junction, an Ohmic contact is formed when $\Phi_m < \Phi_{sc}$.

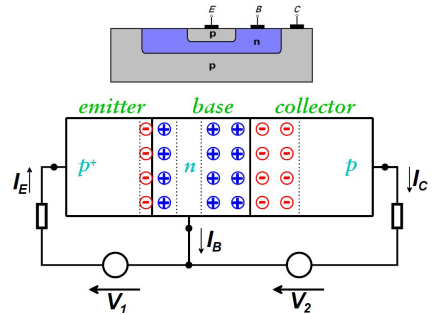


This type of junction conducts well for both senses of applied bias.

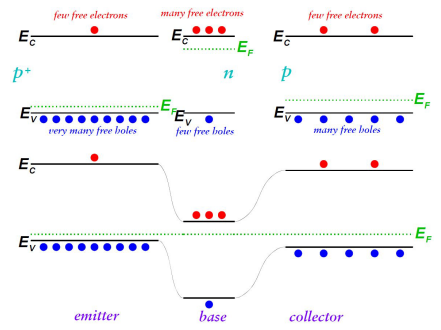
9 The BJT & HBT

9.1 Bipolar Junction Transistor

The bipolar junction transistor (BJT) consists of two pn junctions back to back, with one region common to both.

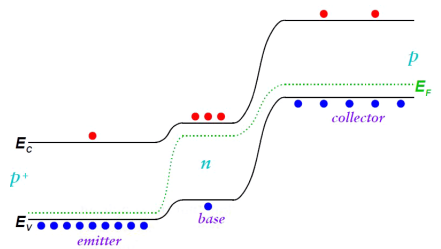


Band bending takes place at each depletion region to ensure that the Fermi energy is constant throughout the device.

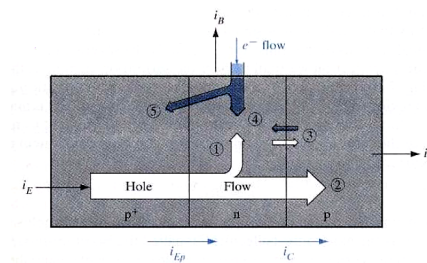


9.2 Applying a Bias

Applying a positive bias to both V_1 and V_2 :



By putting the emitter-base junction into forward bias, a large hole current will flow across the junction from the emitter to the base.



1. Inside the base, the holes find themselves in a region where the majority carriers are electrons, and some holes will recombine with electrons.
2. A large number of holes are injected into the n -region from the emitter and a hole current flows into the collector.
3. Normally only a small reverse bias current will flow as there are few electrons on the p -side and holes on the n -side available.
4. An electron current must be supplied by the base to meet the recombination loss
5. There will be a small electron current in the reverse direction supplied by the base.

If the width of the undepleted region of the base W_b is much less than the diffusion length of the holes $L_h = \sqrt{(D_h \tau_h)}$ then most holes will cross the base without recombination.

9.3 BJT Parameters

Due to *recombination* in the base, not all holes entering the emitter as current I_{Ep} get into the collector and contribute to I_C . The proportion which do is called the *base transport factor*:

$$B = \frac{I_C}{I_{Ep}}$$

The total emitter current, I_E , is made up from the hole component, I_{Ep} , and the electron component, I_{En} , due to electrons injected from the base to the emitter. We define the *emitter injection efficiency*:

$$\gamma = \frac{I_{Ep}}{I_{Ep} + I_{En}}$$

The relation between the total emitter and collection currents is:

$$\frac{I_C}{I_E} = \frac{BI_{Ep}}{I_{En} + I_{Ep}} = B\gamma \equiv \alpha$$

The product $B\gamma$ is defined as the *current transfer ratio*, α .

The relation between the base and collector currents is:

$$\frac{I_C}{I_B} = \frac{BI_{Ep}}{I_{En} + (1-B)I_{Ep}}$$

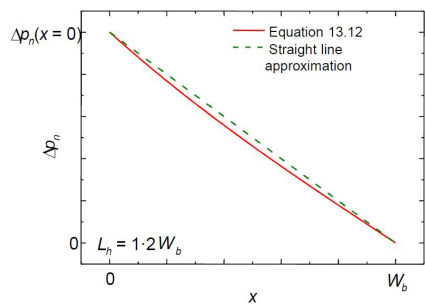
The base to collector current amplification factor is:

$$\beta \equiv \frac{I_C}{I_B} = \frac{B\gamma}{1-B\gamma} = \frac{\alpha}{1-\alpha}$$

9.4 Base Diffusion

The hole concentration as a function of position in the base:

$$\frac{\Delta p_n(x) = \Delta p_n(0) \frac{\exp((W_b - x)/L_h) - \exp((x - W_b)/L_h)}{\exp(W_b/L_h) - \exp(-W_b/L_h)}$$



The result is almost a straight line, with the deviation being due to recombination, which decreases when $W_b \ll L_h$ where $L_h = \sqrt{(D_h \tau_h)}$ is the hole diffusion length.

9.5 Terminal Currents

Assuming that the hole current dominates at these terminals where A is the cross sectional area of the base:

$$I_E \approx \frac{eAD_h \Delta p_n(0)}{L_h} \coth\left(\frac{W_b}{L_h}\right)$$

$$I_C \approx \frac{eAD_h \Delta p_n(0)}{L_h} \operatorname{csch}\left(\frac{W_b}{L_h}\right)$$

$$I_B = I_E - I_C \approx \frac{eAD_h \Delta p_n(0)}{L_h} \tanh\left(\frac{W_b}{2L_h}\right)$$

The base transport factor depends on the transistor design:

$$B = \frac{I_C}{I_{Ep}} = \operatorname{sech}\left(\frac{W_b}{L_h}\right) \approx 1 - \frac{W_b^2}{2L_h^2}$$

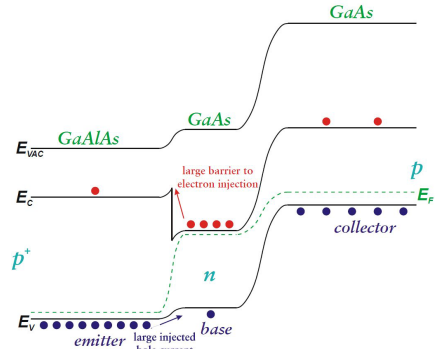
The emitter injection efficiency is:

$$\gamma = \frac{I_{Ep}}{I_{Ep} + I_{En}} = \frac{1}{1 + \left(\frac{D_e}{D_h}\right) \left(\frac{W_b}{L_e}\right) \left(\frac{N_D}{N_A}\right)}$$

9.6 Heterojunction Bipolar Transistor

In order to gain a high γ , we need the doping density in the base to be much less than in the emitter. This increases the resistance of the base, R , and limits the maximum operating frequency.

In the heterojunction bipolar transistor (HBT) we use a different semiconductor with a large band gap for the emitter compared with the base and collector.

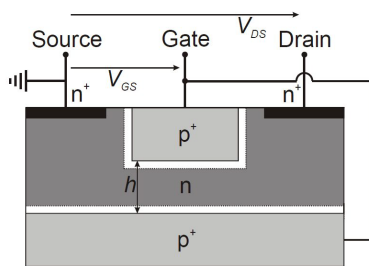


10 The JFET, MESFET & HEMT

10.1 Junction Field Effect Transistors

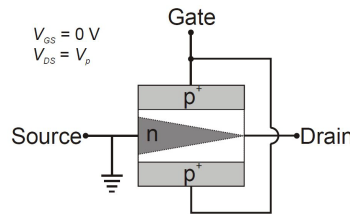
The Field Effect Transistors (FET) are *voltage controlled* and *unipolar* devices.

The Junction Field Effect Transistor (JFET) consists of a conductive channel between a source and a drain (ohmic) contact. The third terminal, the gate, forms a rectifying junction with the channel.



If we maintain $V_{GS} = 0$, as V_{DS} is increased, I_{DS} increases approximately linearly until the depletion regions almost meet (*Ohmic region*).

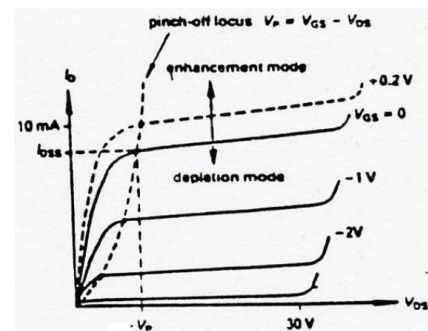
The *pinch-off effect*: The channel width will have decreased and the resistance of the channel, r_d , increased to such an extent that further increase in V_{DS} has no significant effect upon I_{DS} , which remains at its saturated level (*saturated region*).



The voltage, V_p , required to pinch of the channel assuming uniform N_D :

$$-V_p = V_{DS} - V_{GS} = \frac{h^2 e N_D}{8 \epsilon_0 \epsilon_r}$$

The gate voltage, V_{GS} , alters the width of the gate-channel depletion region:



The $I - V$ characteristics of the JFET:

$$I_{DS} = \frac{dhN_D e \mu_e V_p}{L}$$

$$\left\{ \frac{V_{DS}}{V_p} - \frac{2}{3} \left(\frac{V_{GS}}{V_p} \right)^{\frac{3}{2}} + \frac{2}{3} \left(\frac{V_{GS} - V_{DS}}{V_p} \right)^{\frac{3}{2}} \right\}$$

The saturated current can be found at $V_{DS} = V_{GS} - V_p$. When $V_{GS} = 0$:

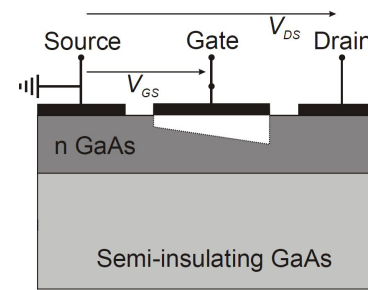
$$I_{DSS} = \frac{-dhN_D e \mu_e V_p}{3L} = \frac{dh^3 N_D^2 e^2 \mu_e}{24L \epsilon_0 \epsilon_r}$$

The small signal mutual conductance in the saturated region is:

$$g_m = \frac{\partial I_{DS}(\text{sat})}{\partial V_{GS}} = \frac{dhN_D e \mu_e}{L} \left\{ 1 - \left(\frac{V_{GS}}{V_p} \right)^{1/2} \right\}$$

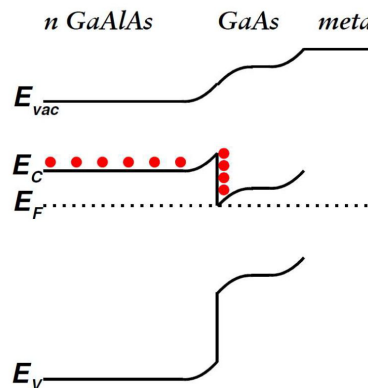
10.2 Metal Semiconductor Field Effect Transistor

The Metal Semiconductor Field Effect Transistor (MESFET) uses a reverse biased Schottky barrier to create a depletion region in a conducting channel instead of a *pn* junction.



10.3 High Electron Mobility Transistor

The High Electron Mobility Transistor (HEMT) is a variant on the MESFET.

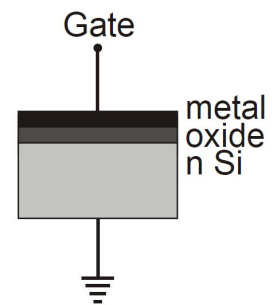


The doped, wide bandgap material provides a source of electrons which accumulate in a two dimensional layer in the GaAs. This is referred to as *two-dimensional electron gas* (2-DEG).

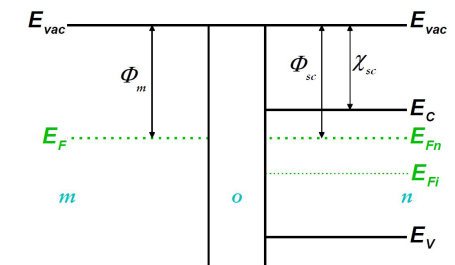
11 The MOSFET

11.1 MOS Capacitor

The MOS capacitor structure:



Consider an n-type semiconductor where there is no difference between the metal and semiconductor work functions, $\phi_m = \phi_{sc}$.

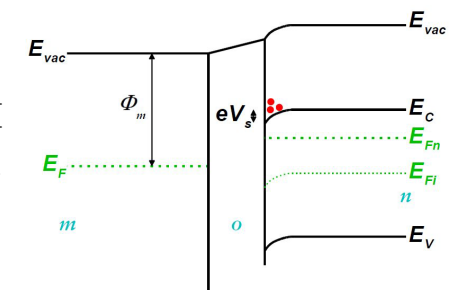


This *flat band* condition occurs when

$$\phi_m - \phi_{sc} = \phi_m - \left[\chi_{sc} + \frac{E_C}{2} - (E_{Fn} - E_{Fi}) \right] = 0$$

11.2 Accumulation

When a positive bias is applied to the gate, the *accumulation* of electrons will cause the conduction and valence bands in the semiconductor to bend down and a *surface potential*, V_s , will exist.



11.3 Depletion

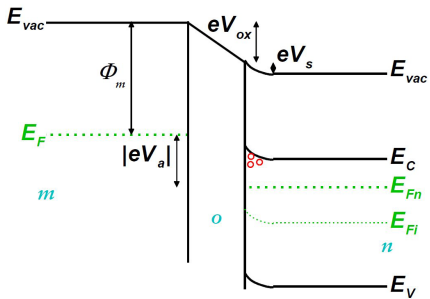
If a negative bias is applied to the gate, the electrons are repelled away from the surface by the applied electric field and we have a *depletion region*.

If V_s is the surface potential of the semiconductor then

$$V_s = \epsilon w = \frac{eN_D w^2}{2\epsilon_0 \epsilon_r}$$

The voltage applied to the metal gate is V_a and the voltage drop across the oxide is V_{ox} must satisfy:

$$-V_a = V_{ox} + V_s$$

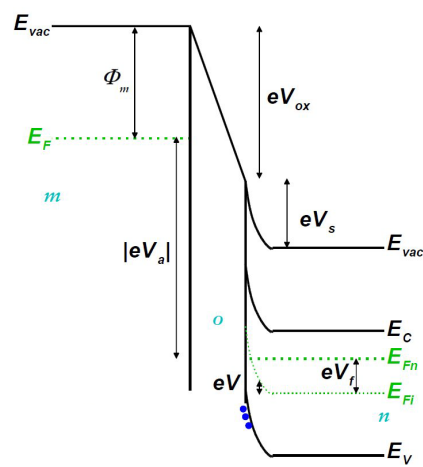


11.4 Inversion

If we continue to increase the magnitude of the negative voltage applied to the gate until we will have as many holes at the interface as we have electrons in the bulk.

Strong Inversion: Assuming that $N_C = N_V$, this occurs when the Fermi level at the interface is as close to the valence band as it is to the conduction band in the bulk.

$$eV_f = E_{Fn} - E_{Fi} = E_g - kT \ln \left(\frac{N_C}{N_D} \right) = kT \ln \left(\frac{N_D}{n_i} \right)$$



11.5 Surface Space Charge

The electric field at the interface with the oxide layer ($x = 0$) where the surface potential is V_s is:

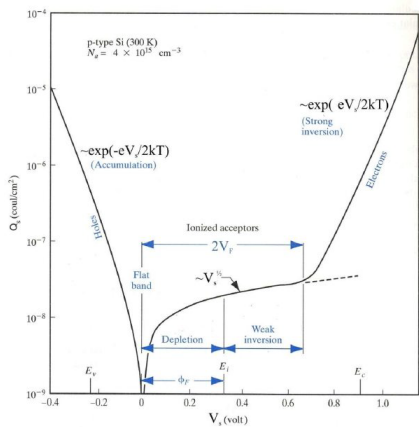
$$E_s = \left(\frac{\sqrt{2kT}}{eL_D} \right) \left[\frac{p_0}{n_0} \left\{ \exp \left(\frac{-eV_s}{kT} \right) + \frac{eV_s}{kT} - 1 \right\} + \left\{ \exp \left(\frac{eV_s}{kT} \right) - \frac{eV_s}{kT} - 1 \right\} \right]^{1/2}$$

The *Debye screening length*, L_D , is the length scale over which electric fields in the semiconductor are screened.

$$L_D = \left(\frac{\epsilon_0 \epsilon_r kT}{e^2 n_0} \right)^{1/2}$$

From the Gauss Law of Electrostatics, the surface space charge per unit area is:

$$Q_s = -\epsilon_0 \epsilon_r E_s$$



If C_{ox} is the capacitance per unit area of the insulator, then the voltage drop across the insulator must be:

$$V_{ox} = -\frac{Q_s}{C_{ox}}$$

Strong inversion begins at a surface potential of:

$$V_s(\text{inv}) = 2V_f = \frac{2kT}{e} \ln \left(\frac{N_D}{n_i} \right)$$

The maximum depletion width occurs at the onset of strong inversion:

$$w_{\text{max}} = 2 \left(\frac{\epsilon_0 \epsilon_r V_f}{eN_D} \right)^{1/2}$$

Deep depletion: If the voltage applied to the metal is varied more rapidly than the time required to produce the holes, then the depletion region can be extended much further (quasi-equilibrium condition).

The time required for strong inversion to occur:

$$\tau_{\text{inv}} = \frac{en_0 t_{\text{inv}}}{p_0 e v_s}$$

11.6 Threshold Voltage

The charge per unit area in the depletion region at strong inversion is:

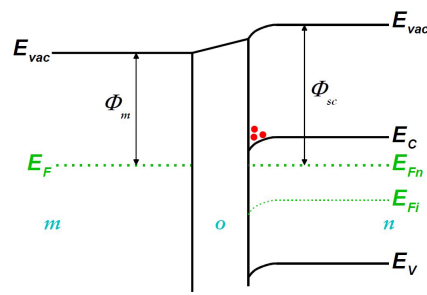
$$Q_d = eN_D w_{\text{max}} = 2 \left(\epsilon_0 \epsilon_r e N_D V_f \right)^{1/2}$$

If the oxide capacitance per unit area is C_i , then the threshold voltage dropped across both the oxide and the depleted region is:

$$V_T = -\frac{Q_d}{C_i} - 2V_f$$

11.7 Real MOS Systems

In general, there will be a difference between the work functions of the metal and semiconductor. A built in potential will form between the semiconductor and the metal, much of which will be dropped across the oxide and a proportion will be taken up in band bending in the semiconductor.



For $\phi_m < \phi_{sc}$, there is an additional accumulation of electrons at the oxide-semiconductor interface.

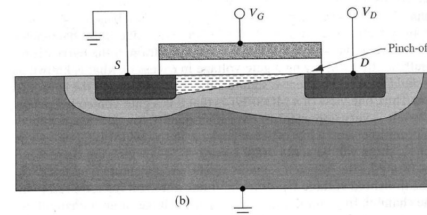
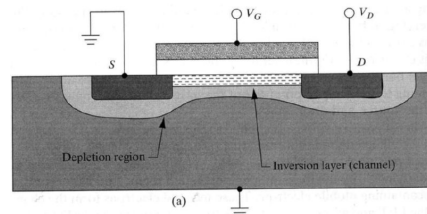
In order to achieve a flat band condition, we need to apply a voltage to the metal gate of:

$$V_{FB} = \frac{\phi_m - \phi_{sc}}{e}$$

Extra charges introduced to the system by the oxide fall into four types:

1. Interface trapped charges, Q_{it}
2. Fixed oxide charges, Q_f
3. Oxide trapped charges, Q_{ot}
4. Mobile ionic charges, Q_m

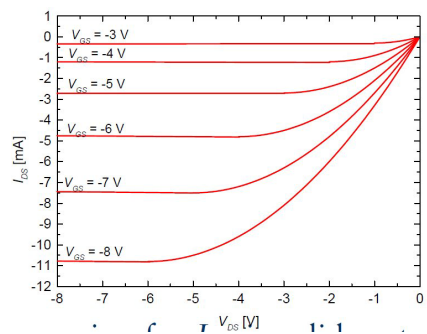
11.8 MOSFET Characteristics



If we apply a voltage V_{DS} to the drain, the drain source current, I_{DS} is then

$$I_{DS} = -\frac{C_i \mu_{hFE} W}{L} \left[(V_{GS} - V_T) V_{DS} - \frac{V_{DS}^2}{2} \right]$$

μ_{hFE} is the *field effect mobility* of carriers in the channel.



This expression for I_{DS} is valid up to saturation current, I_{DSsat} , where the channel has been pinched off, and no more current can be forced through the channel.

The saturation current is:

$$I_{DS}(\text{sat}) = -\frac{C_i \mu_{hFE} W (V_{GS} - V_T)^2}{2L}$$

In the linear region, the small signal channel conductance and mutual transconductance can be obtained:

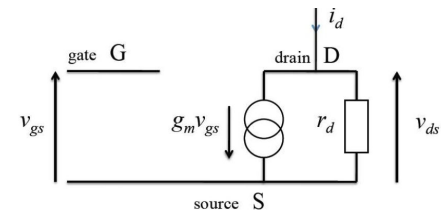
$$g_s = \frac{\partial I_{DS}}{\partial V_{DS}} = \frac{C_i \mu_{hFE} W}{L} (V_{GS} - V_T)$$

$$g_m = \frac{\partial I_{DS}}{\partial V_{GS}} = \frac{C_i \mu_{hFE} W}{L} V_{DS}$$

The mutual transconductance in the saturated region is given by:

$$g_m = \frac{\partial I_{DS}(\text{sat})}{\partial V_{GS}} = \frac{C_i \mu_{hFE} W}{L} (V_{GS} - V_T)$$

The small signal equivalent circuit for the MOSFET:



(The End)