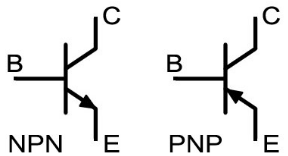
1 Bipolar Transistor

1.1 Bipolar Junction Transistors



BJTs have low input impedance to base. The output is collector current, which depends on base-emitter voltage (and current).

1.2 The Ebers-Moll Model

The collector current I_C is related to the base-emitter voltage V_{BE} by:

$$I_C = I_s\left[e^{\frac{qV_{BE}}{kT}} - 1\right] \approx I_s e^{\frac{q_0 V_{BE}}{kT}} = I_s e^{\frac{V_{BE}}{V_t}}$$

I_s is a constant determined by the transistor construction. $V_t = kT/q \sim 25\text{mV}$ at room temperature.

This is similar in form to the pn diode equation where for the base-emitter diode:

$$I_B = I_s'\left[e^{\frac{qV_{BE}}{kT}} - 1\right]$$

The current gain of the transistor $I_s/I_s' = h_{fe}$ is approximately constant (over a limited range)

$$I_C = I_B h_{fe}$$

h_{fe} is often around $100-500$ for low current, small signal devices.

1.3 Emitter Resistance

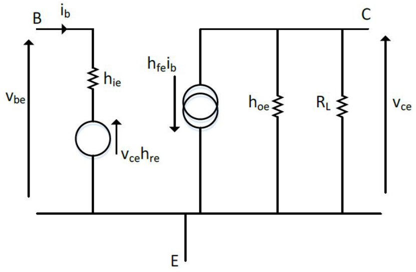
There is an internal emitter resistance associated with BJTs which depends on the current flowing through the device. $I_C \simeq I_E$ as $I_B \ll I_{C,E}$.

$$r_e = \frac{\partial V_{BE}}{\partial I_C} \approx \frac{dV_{BE}}{dI_C} = \left(\frac{dI_C}{dV_{BE}}\right)^{-1} \approx \frac{V_t}{I_C}$$

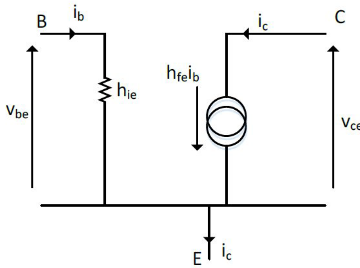
Hence, $r_e \approx (25/I_C)\Omega$ with I_C in mA

1.4 The Small Signal Model

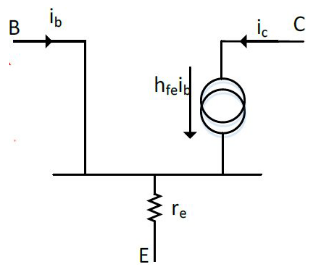
For a typical BC108 npn transistor with $I_C = 2\text{mA}$ and $V_{BE} = 0.6\text{V}$, the transistor can be modelled with the following circuit:



We can ignore h_{oe} ($\sim 6\%$) and h_{re} ($\sim 5\%$) and the small signal model can be simplified to:



Consider the equivalence of this base resistance model to an emitter resistance model:



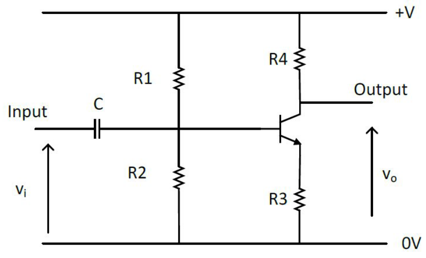
For equivalence of V_{be}:

$$r_e = \frac{h_{ie}}{h_{fe}} = \frac{V_t}{I_C}$$

2 Bipolar Transistor Amplifier Design

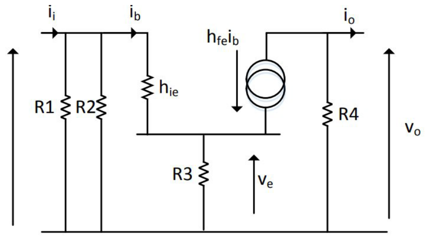
2.1 Amplifier Circuit

Consider the following general amplifier circuit:



Function of passive components:

- C is the ac signal coupling capacitor: isolates the dc bias levels between stages
- R_1 and R_2 form a voltage source to provide a d.c. base bias current
- R_3 provides negative feedback to the base bias current to stabilise the bias point
- R_4 is the collector output load resistor: changes in collector current create an output voltage swing.



$$i_b = \frac{v_i}{h_{fe}R_3 + h_{ie}}$$

The input impedance is:

$$\begin{aligned} R_i &= R_1 \| R_2 \| (h_{fe}R_3 + h_{ie}) \\ &= R_1 \| R_2 \| h_{fe}(R_3 + r_e) \end{aligned}$$

Gain of the amplifier:

$$\text{Gain} = \frac{-h_{fe}R_4}{(h_{fe}R_3 + h_{ie})} = \frac{-R_4}{R_3 + r_e}$$

The output impedance is R_4.

$$v_e = \frac{R_3}{R_3 + r_e} v_i$$

2.2 Amplifier Design

Steps to design a simple bipolar transistor amplifier:

1. Find the number of stages required at 20dB *power* gain per transistor.
   Power gain in dB:
   $$10\log_{10}\left(\frac{P_{out}}{P_{in}}\right)$$
2. Work out the required voltage gain from each stage by dividing the total gain required equally between the n stages.
   $$V_{rms} = \frac{1}{2\sqrt{2}} V_{pp}$$
   Assuming the output and input impedances are matched, we lose 50% of the signal volrage at each coupling.
   Total gain product:
   $$\text{Gain} = \frac{1}{(0.5)^{n+1}} \frac{V_{out}}{V_{in}}$$
   Divided between n stages $=$ $\sqrt[n]{\text{Gain}}$ per stage.
3. Set the ratio of input/output impedance for each stage.
   $$\sqrt[n]{\frac{R_S}{R_L}}$$
4. Choose R_4's from output impedances required, working back from output.
   $$R_{out} = R_4$$
5. Choose R_3's from gains required (include a safety margin).
   $$\text{Gain} \approx -\frac{R_4}{R_3}$$
6. Choose R_2's and R_1 's from input impedances required and $V_{BE} = 0.6\text{V}$.
   Rules of thumb:
   $$V_C = \frac{V_S}{2}$$
   $$V_E \sim \frac{V_S}{20}$$
   $$V_B = (V_E + 0.6)$$
   From the small signal model:
   $$R_{in} \approx R_1 \| R_2 \| R_3 h_{fe}$$
   Take $h_{fe} = 250$. Choose $R_2 \approx 2R_{in}$.
7. Select transistors for high h_{fe}, voltage, current and power ratings
8. Check effect of low h_{fe}, particularly for higher current stage - re-jig values as required.

The series capacitors for ac signal coupling between stages are selected in combination with the input tand output impedances to give a low frequency roll-off:

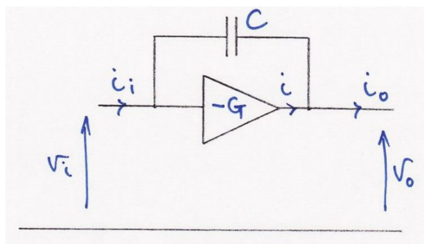
$$f_{3dB} = \frac{1}{2\pi\left(R_{out(n)} + R_{in(n+1)}\right)C}$$

The action of R_3, of providing *negative feedback*, improves the stability of the circuit but reduces the gain.

3 Transistor Frequency Response

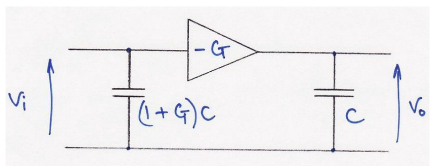
3.1 Miller Effect

Consider an (inverting) amplifier with some capacitance between its input and output.

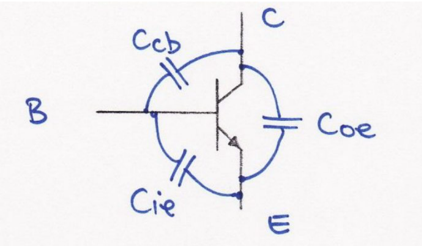


The input capacitance is equivalent to $(1+G)C$. The magnification of capacitance is known as the *Miller Effect*.

The output capacitance is equivalent to $\left(1+\frac{1}{G}\right)C \approx C$.



3.2 Transistor Capacitance



c_{ie} ($\sim 10-100$pF) is predominantly due to a forward biased pn junction.

c_{cb} and c_{oe} ($\sim 1-10$pF) are lower parasitic values through a reverse biased junction.

The base-emitter capacitance c_{ie} depends on the current passing through the transistor.

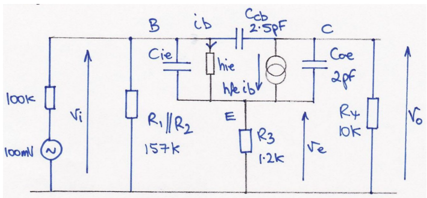
$$c_{ie} = kI_E \approx kI_C$$

Since $r_e = 0.025/I_C$, f_t is given for transistors representing useful bandwidth independent of current:

$$f_t = \frac{1}{2\pi C_{ie} r_e} \approx \frac{1}{0.05\pi k}$$

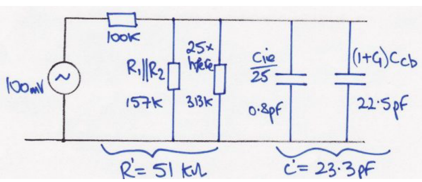
The main gain and frequency characteristics of a transistor may be described by 2 parameters, h_{fe} and f_t.

For the first stage of the audio amplifier, the small signal model is:



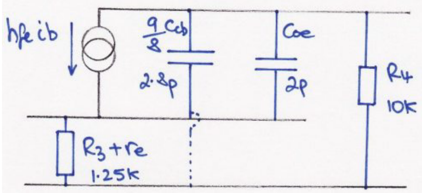
The equivalent base-emitter impedances referred to ground are $\times \frac{v_i}{v_i - ve}$.

The equivalent input circuit becomes:



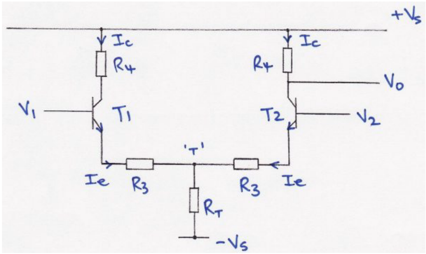
$$f_{-3\text{dB}} = \frac{1}{2\pi R'C'}$$

The output circuit is:



3.3 Differential Amplifier

The differential amplifier is sometimes called a *long-tailed pair*.



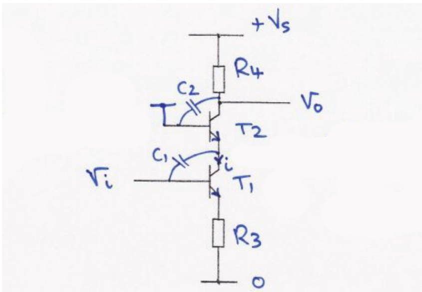
Differential gain: If we apply a differential input voltage v_i between the 2 bases such that: base $1 \Rightarrow +v_i/2$ and base $2 \Rightarrow -v_i/2$, therefore $\Delta I_e = v_i/2R_3 = \Delta I_c$ since the emitter follows the base voltage change and the T point voltage does not change.

The differential gain $= \frac{R_4}{2R_3}$.

Common mode gain: If we apply a common mode ie. equal input voltage to each base then, base 1 and base $2 \Rightarrow +v_i$, therefore $\Delta I_e = v_i/(2R_T + R_3)$ since the emitters follow the base voltage change and the impedance of the emitters to ground is through the pair of R_3's and R_T.

The common mode gain $= \frac{R_4}{2R_T + R_3}$.

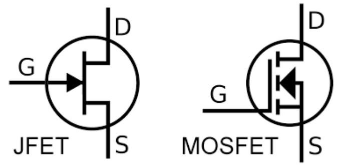
3.4 The Cascode Circuit



In this circuit the input transistor T_1 has a collector voltage fixed at $3\text{ V} - 0.6\text{ V} = 2.4\text{ V}$, hence there is no Miller effect with C_1 at the input.

4 Field Effect Transistors

4.1 FET Characteristics

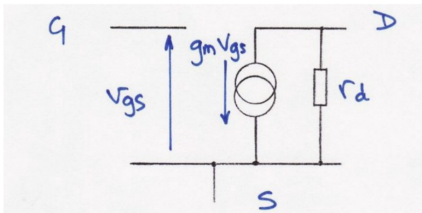


When the gate to source voltage V_{GS} rises above a threshold offset value V_{th}, then the transistor begins to conduct a current I_D between its drain and source with a voltage V_{DS} across it.

There are 3 regimes of interest:

1. $(V_{GS} - V_{th}) < 0$, $\quad I_D = 0$ ie. the device is *off*.
2. $(V_{GS} - V_{th}) \geq V_{DS}$, $I_D = V_{DS} 2K\left[(V_{GS} - V_{th}) - V_{DS}/2\right]$ ie. it's a voltage controlled resistor (linear)
3. $(V_{GS} - V_{th}) \leq V_{DS}$, $I_D = K\left(V_{GS} - V_{th}\right)^2$ ie. it's a voltage controlled current source (saturation)

4.2 Small Signal Model

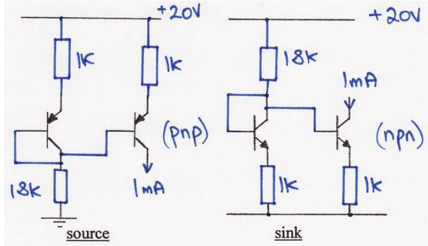


This applies to saturation regime where $(V_{GS} - V_{th}) < V_{DS}$

4.3 Current Sources

Simple FET constant current diodes are available to supply a constant current with pre-selected values.

For more accurate current sources, a current mirror can be used where a reference current is drawn through one of a matched pair of transistors to create a V_{BE} corresponding to that current.

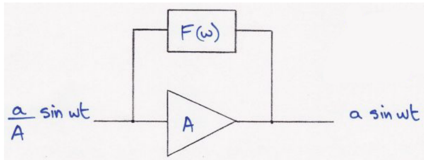


5 Oscillators

5.1 Positive Feedback

Consider an amplifier with positive feedback including a filter in the feedback path. The circuit will sustain stable oscillations when the gain around the amplifier/ feedback filter loop is unity and the phase shift zero.

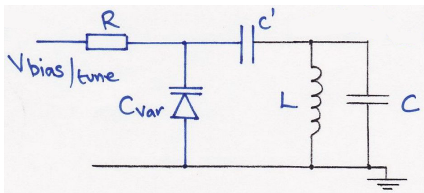
$$|AF(\omega_0)| = 1$$
$$\angle AF(\omega_0) = 0 \text{ or } 360°$$



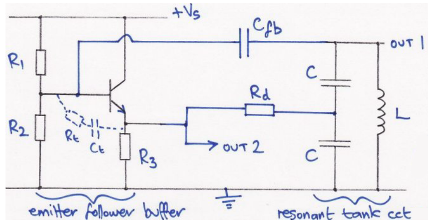
5.2 RF Oscillators

We can tune a radio oscillator using the *Voltage Controlled Oscillators* (VCOs) based on the variable capacitance of a reversed biased pn diode (varactors).

By including the capacitance of a varactor in an LC tank circuit, the frequency can be voltage tuned by a factor of 2 or more.



The Colpitts Oscillator: We use an emitter follower (gain ~ 1) driving the LC network (gain ~ 2) to create a positive feedback loop.



There are 2 convenient points to take the output from:

1. Across L for minimum distortion, but only for high impedance loads.
2. Across R_3 at the emitter for higher output power and load isolation, but higher harmonic distortion without careful trimming.

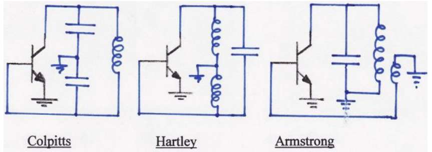
In radio circuits, power is used to describe signal levels in dBm (decibels relative to 1mW).

$$\text{dBm} = 10\log_{10}\left(P/10^{-3}\right)$$

Steps to design an oscillator circuit for a radio tuner:

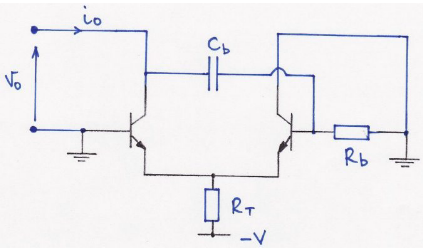
1. Check power level $\leq 60\%$ of supply (5V).
2. Select transistor and set d.c. bias resistors.
   Choose low power NPN transistor with $f_t >>$ required frequency. Set base bias to give $V_E \approx 1/2$ supply and $V_B = 0.6 + V_E$.
   Choose $R_3 \approx 1.5R_L$ to $2R_L$. Choose values of R_1 and $R_2 \sim h_{fe}(R_3||R_L)$.
3. Select LC and varactor.
   Total C tuning ratio is:
   $$\frac{C_{max}}{C_{min}} = \left(\frac{f_{max}}{f_{min}}\right)^2$$
   Choose big C_{fb} to block d.c., low impedance at signal frequency.
   Select a varactor with C_{var}.
   $$C_{total} = C_{var} + C/2$$
   $$L = \frac{1}{(2\pi f_{mid})^2 C_{mid}}$$
4. Select LC network drive resistor to be 1/5 of all resistances parallel to L.
   $$R_d \approx \frac{1}{5}(R_1||R_2||\omega LQ||h_{fe}R_3||h_{fe}R_L)$$
   The quality factor $Q \sim 50$

The Hartley and Armstrong circuits:

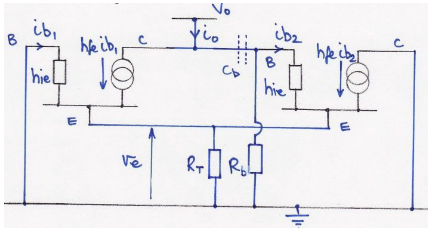


5.3 Negative Impedance Oscillator

In the negative impedance oscillator, a pair of (fast) transistors are configured to give an apparent negative resistance which compensates for the circuit resistive losses and the resonant LC oscillations do not die away.

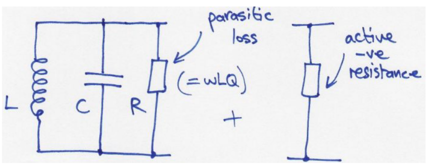


The small signal model equivalent:



$$Z_0 = R_b || -2r_e$$

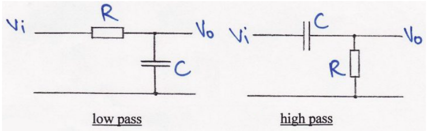
Hence we can make a negative impedance based oscillator circuit by adding a suitable tuned LC circuit and cancelling out the resistive parasitic component of the LC combination.



The oscillation will grow until the voltage swing across the transistors prevents them from making an effective negative impedance and a steady oscillation is observed.

6 Filters

6.1 Passive RC Filter



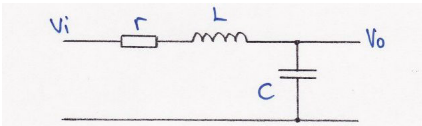
For low pass filter:

$$\frac{v_o}{v_i} = \frac{1}{1+j2\pi fCR}$$

For high pass filter:

$$\frac{v_o}{v_i} = \frac{1}{1-\frac{j}{2\pi fcR}}$$

6.2 Resonant LC Circuit



The resonant angular frequency is given when:

$$w_0^2 = \frac{1}{LC}$$

The quality factor is:

$$Q = \frac{1}{\omega_0 Cr} = \frac{\omega_0 L}{r}$$

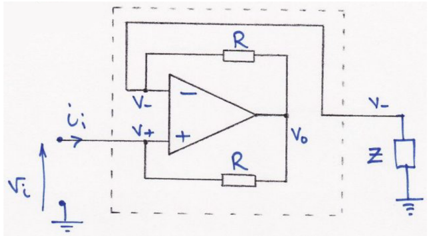
For the resonant LC circuit:

$$\frac{v_o}{v_i} = \frac{1}{\left(1-\omega^2 LC\right)+j\omega Cr} = \frac{1}{1-\left(\frac{w}{w_0}\right)^2+\frac{j}{Q\left(\frac{w}{w_0}\right)}}$$

If $\omega = \omega_0$, $\left|\frac{v_o}{v_i}\right| = Q \sim 100$.

If $\omega = 1.1\omega_0$, $\left|\frac{v_o}{v_i}\right| \approx 5$.

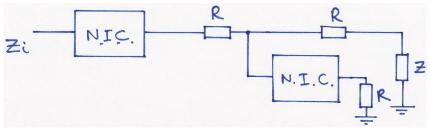
6.3 Negative Impedance Converter (NIC)



$$Z_i = \frac{v_i}{i_i} = -Z$$

If Z is a resistor R, then Z_i is $-R$. If Z is a capacitor then $Z_i = j/\omega C$ (not an inductor as the frequency dependence is reciprocal).

The gyrator:



$$Z_i = -\left(R+\frac{-R(R+Z)}{-R+R+Z}\right) = \frac{R^2}{Z}$$

If Z is a capacitor, then $Z_i = j\omega CR^2$ ie. an inductor with $L = CR^2$.

6.4 Filter Polynomials

Consider 3 types of (n-pole only) filters:

1. *Bessel*

$$\left|\frac{v_o}{v_i}\right| \approx 10^{-\frac{3}{20}\left(\frac{f}{f_c}\right)^2}$$

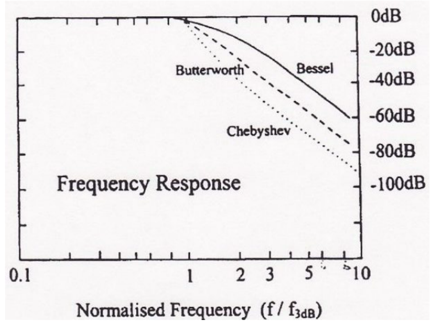
2. *Butterworth*

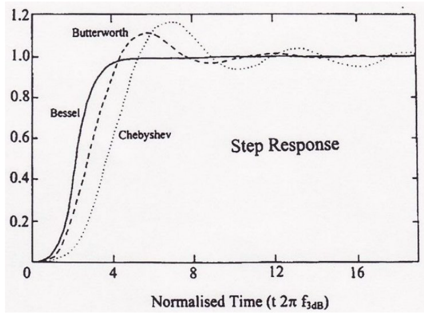
$$\left|\frac{v_o}{v_i}\right| = \left[1+\left(\frac{f}{f_c}\right)^{2n}\right]^{-\frac{1}{2}}$$

3. *Chebyshev*

$$\left|\frac{v_o}{v_i}\right| = \left[1+\epsilon^2 c_n^2\left(\frac{f}{f_c}\right)\right]^{-\frac{1}{2}}$$

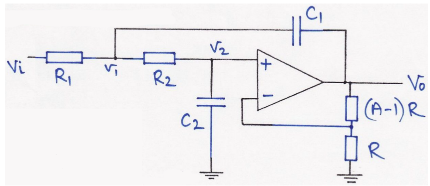
f_c is the -3dB cut-off frequency, ϵ is between 0 and 1 and $c_n\left(\frac{f}{f_c}\right)$ is the Chebyshev polynomial of order n for $\frac{f}{f_c}$.





6.5 Voltage-Controlled Voltage Source (VCVS)

The low pass 2-pole VCVS filter:



For convenience, $C_1 = C_2 = C$ and $R_1 = R_2 = R$.

$$\left|\frac{v_o}{v_i}\right| = A\left[1+\left(\frac{\omega}{\omega_n}\right)^2 B+\left(\frac{\omega}{\omega_n}\right)^4\right]^{-\frac{1}{2}}$$

The resonant frequency $\omega_n = \frac{1}{CR}$ and the constant $B = (3-A)^2 - 2$. If $B = 0$, this is a Butterworth filter.

Higher orders can be realised by cascading several stages (each stage gives 2 poles).

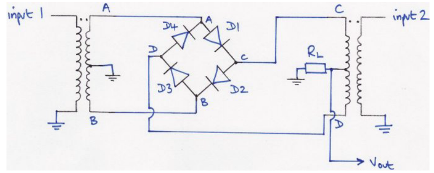
f_{-3dB} desired overall $= 1/(2\pi f_n CR)$ for each stage. For high pass sections, take the reciprocal of f_n as the frequency scaling factor.

7 Mixers

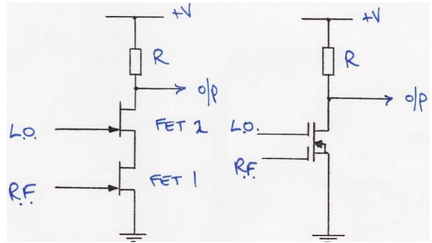
7.1 Phase Detectors

The output signal of a mixer contains sum and difference frequencies of the inputs.

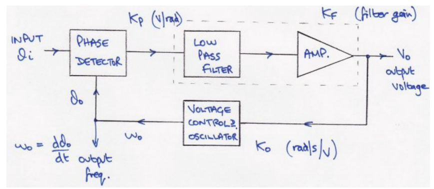
Double balanced mixer:



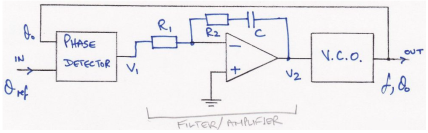
The cascode or dual-gate MOSFET circuits:



7.2 The Phase Locked Loop (PLL)



The phase detector (ie. mixer) compares the phase of the input with the VCO output and produces a phase error signal with a dc component. This signal is filtered and fed to the VCO such that the frequency and phase of the VCO output locks to that of the input signal and will track with it.



All oscillating terms implicitly multiplied by $e^{j\omega t}$.

$$\theta_0 \equiv e^{j(\omega t+\theta)}$$
$$\theta_{\text{ref}} = e^{j\omega t}$$

For the phase detector:

$$V_1 = K_p(\theta_0 - \theta_{\text{ref}})$$

For the VCO:

$$2\pi f = \frac{d\theta_0}{dt} = j\omega\theta_0 = K_o V_2$$

If $\omega_n^2 = \frac{K_o K_p}{CR_1}$ and $\zeta = \frac{\omega_n CR_2}{2}$ then:

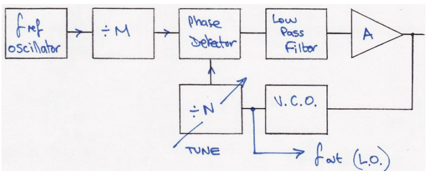
$$\frac{\ddot{\theta}}{\omega_n^2} + \frac{\dot{\theta}2\zeta}{\omega_n} + \theta = \theta_{\text{ref}} + \frac{2\zeta}{\omega_n}\dot{\theta}_{\text{ref}}$$

Note that ω is the frequency of relative phase swing or variation between the input signal and VCO frequency once the loop is locked. The lock-up time is give approximately by:

$$t \approx \frac{4\delta\omega^2}{B^3}$$

$\delta\omega$ is the initial frequency difference and B is the loop bandwidth $= K_o K_p K_F$.

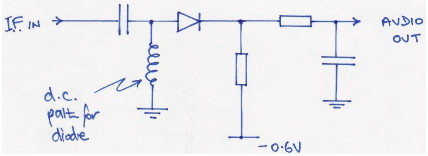
FM radio LO with quartz crystal reference:



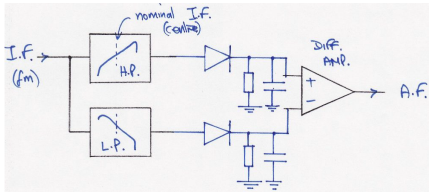
$$\frac{f_{\text{ref}}}{M} = \frac{f_{\text{out}}}{N}$$

7.3 Demodulation Circuits

Amplitude modulation:



Frequency and phase modulation: Band edge filters (slope detectors) can convert FM to AM. A pair of matched inverse linear slope filters can be used for a linear response.



8 Transmission Lines

8.1 Characteristic Impedance

For a transmission line with L = inductance per unit length and C = capacitance per unit length, the wave velocity is given by:

$$v = \frac{1}{\sqrt{LC}}$$

The *characteristic impedance* is:

$$Z_0 = \sqrt{\frac{L}{C}}$$

The phase constant is:

$$\beta = 2\pi/\lambda$$

The voltage/field reflection coefficient is:

$$\rho = \left|\frac{Z_L - Z_0}{Z_L + Z_0}\right|$$

The power reflection coefficient $P = \rho^2$.

The input impedance for a length of transmission line terminated with a given impedance is:

$$Z_{\text{in}} = Z_0\left(\frac{Z_L + Z_0 j\tan\beta L}{Z_0 + Z_L j\tan\beta L}\right)$$

8.2 Controlled Impedance

For a Microstrip line with width w and dielectric thickness d, the capacitance per unit length of the line is estimated by:

$$C = (w + 2d)\frac{\epsilon_0\epsilon_r}{d}$$

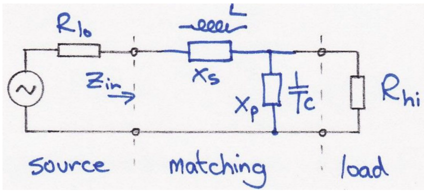
The speed of light in the dielectric is:

$$v = \frac{1}{\sqrt{\epsilon_0\epsilon_r\mu_0}} = \frac{c_0}{\sqrt{\epsilon_r}}$$

The characteristic impedance of the Microstrip is given by:

$$z_0 = \frac{1}{vC} = \frac{d}{w+2d}\cdot\frac{1}{c_0\epsilon_0\sqrt{\epsilon_r}}$$

8.3 Impedance Matching



X_p shunts R_{hi} and X_s series R_{lo}.

$$Z_{\text{in}} = X_s + \frac{R_{\text{hi}}X_p}{R_{\text{hi}} + X_p}$$
$$= j\omega L + \frac{R_{\text{hi}} - j\omega CR_{\text{hi}}^2}{\left(1 + \omega^2 c^2 R_{\text{hi}}^2\right)}$$

If $L = \frac{CR_{\text{hi}}^2}{(1+\omega^2C^2R_{\text{hi}}^2)}$ then:

$$Z_{\text{in}} = \frac{R_{\text{hi}}}{\left(1 + \omega^2C^2R_{\text{hi}}^2\right)}$$

With $\omega^2 = \frac{CR_{\text{hi}}^2 - L}{LC^2R_{\text{hi}}^2}$:

$$Z_{\text{in}} = \frac{R_{\text{hi}}}{CR_{\text{hi}}^2/L} = \frac{L}{CR_{\text{hi}}}$$

We can make $\mathfrak{Re}\left(R_{\text{hi}}\|X_p\right) < R_{\text{hi}}$ and cancel any $\mathfrak{Im}\left(R_{\text{hi}}\|X_p\right)$ with a series X_s. The sharpness of the response is given by the Q factor.

$$Q = \frac{R_{\text{hi}}}{X_p} = \frac{X_s}{R_{\text{lo}}} = \sqrt{\frac{R_{\text{hi}}}{R_{\text{lo}}} - 1}$$

The effective Q_f is half the filter Q. The half power bandwidth B is given by:

$$B = \frac{2f_0}{Q} = \frac{f_0}{Q_f}$$

9 Antennas

9.1 The Ideal Dipole

The total power radiated from an ideal dipole is:

$$P_r = 40\pi^2 I^2\left(\frac{\Delta z}{\lambda}\right)^2 = \frac{1}{2}I^2 R_r$$

R_r is the radiation resistance.

$$R_r = 80\pi^2\left(\frac{\Delta z}{\lambda}\right)^2$$

The radiation efficiency is given by:

$$e = \frac{P_r}{P_{\text{in}}}$$
$$P_{\text{in}} = \frac{1}{2}I^2(R_r + R_{\text{ohmic}})$$

Gain is the maximum power radiated per unit area divided by power per unit area from isotropic antenna:

$$G = \frac{P_{\text{max}}/\text{unit area}}{P_{\text{in}}/4\pi r^2}$$

The directivity gives an indication of how directional an antenna is:

$$D = \frac{G}{e}$$

The power delivered into a matched load by an antenna is the product of effective aperture A_e and the power density in incident radio wave.

Antenna equation:

$$G = \frac{4\pi A_e}{\lambda^2}$$

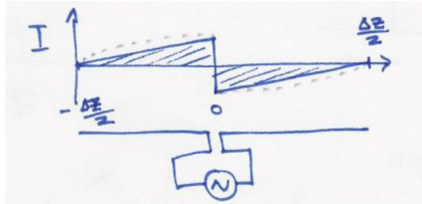
The current is carried only by an outer layer of thickness, δ, within the surface of a conductor where the skin depth, δ, is given by:

$$\delta = \sqrt{\frac{2}{\omega\mu\sigma}}$$

μ is the magnetic permeability and σ is the metal conductivity ($\sigma = 1/\rho$, resistivity).

9.2 Practical Antennas

Any antenna can be made up by integrating the ideal dipole, with appropriate current distributions, over the antenna structure.



$$R_r = 20\pi^2\left(\frac{\Delta z}{\lambda}\right)^2$$

For cosine distribution:

$$R_r = 30\pi^2\left(\frac{\Delta z}{\lambda}\right)^2$$

Consider an antenna with length L and diameter d.

$$R_{\text{ohmic}} = \frac{\rho L}{A} = \frac{\rho L}{\pi d\delta}$$
$$e = \frac{R_r}{R_r + R_{\text{ohmic}}}$$

(The End)